

ANALYSIS OF EQUIPMENT LOADS ON GEOCOMPOSITE LINER SYSTEMS ¹

David J. Kerkes, Ph.D., P.E.

Consulting Geotechnical (Civil) Engineer

1311 Mustang Trail • Kingwood, Texas 77339 USA

dkerkes@geotechconsultant.com • <http://www.geotechconsultant.com>

ABSTRACT

The loads imposed on geosynthetics during installation are frequently the most severe that the materials will experience during their service life; however, they are among the most difficult to evaluate. This paper examines the loads imposed by track mounted and rubber tired vehicles spreading soil over the liner system during construction and proposes an analysis that uses three-dimensional sliding blocks for computing a factor of safety for the liner system under such loads, which takes into account the effect of the soil cover between the equipment and potential failure surface, as well as the effect of tensile forces in components of the liner above that surface. The solution algorithm, which is presented in some detail, can be executed using standard spreadsheet software. The limitations of the method are also discussed, and suggestions are made for using the method in light of the complex stress-strain behavior of composite liner systems.

INTRODUCTION

The realization that equipment loads need to be considered in the design of geocomposite liners is by no means a new idea and has been treated by McKelvey and Deutsch (1991), Druschel and Underwood (1993), McKelvey (1994), Corcoran and McKelvey (1995), as well as Koerner and Soong (1998). The methods that the author is aware of consider the problem in terms of two (an active and passive) sliding blocks, with the active block consisting of the entire slope and the passive block located at the toe. The positive effect of the passive block is sometimes referred to as "toe buttressing," and the equipment load is simply added to the active block. The limitation imposed by the location of the passive block, inclination of its base, and side force inclination between blocks, seems to preclude all but one type of potential failure surface. In this way, the passive block provides support to the active block irrespective of where the equipment may be located on the slope (i.e., a localized failure is not considered possible at any distance upslope of the passive block). The contention is made in this paper that a localized failure is possible at some point above the toe of the slope and that the forces in the immediate vicinity of the equipment need to be examined in this regard. Furthermore, the forces associated with equipment loading are very much a function of the exact type of equipment working on the slope, and some of the specifics are discussed. A sliding block analysis is proposed that considers three (active, central, and passive) blocks above the potential failure surface, and the force associated with geosynthetic reinforcement is included in the analysis.

While the equations presented herein may initially appear to be complex, they are in fact quite straightforward and lend themselves well to solution using commercially available spreadsheet software.

¹ Proceedings of the Geosynthetics '99 Conference, IFAI, Boston, MA, April 1999, pp 1043-1054. Reprinted by the author with the permission of The Industrial Fabrics Association International (IFAI).

It is the availability of such software, with its built-in mathematical operators and interactive cells, that now enables engineers to formulate solutions of increasing complexity without the need to develop a program in a specific computer language, such as FORTRAN or BASIC. The nature of the solution algorithm will be briefly discussed; however, anyone comfortable with spreadsheets and macros will readily see how to develop an electronic worksheet to solve the equations. To assist the reader, the layout of a spreadsheet developed for this purpose is provided as a numerical example, which includes appropriate macro commands in Quattro-Pro.

LOADS IMPOSED BY CONSTRUCTION EQUIPMENT

The loads imposed by construction equipment depend on both the type of equipment and direction of travel on the slope. An analysis of these loads needs to consider the exact pieces of equipment being proposed for use in construction of the liner system, thus making the analysis quite site specific. While the subject of loads imposed by construction equipment has been treated in the literature, it appears that all of the forces are not always being considered. Let us first examine the case of a track mounted bulldozer spreading soil cover in the upslope direction as illustrated in Figure 1. The forces that must be overcome as the dozer moves up the slope are transferred to the underlying soil and geocomposite liner system by the tracks of the bulldozer. While the weight of the equipment (W_{eq}) is an obvious force that

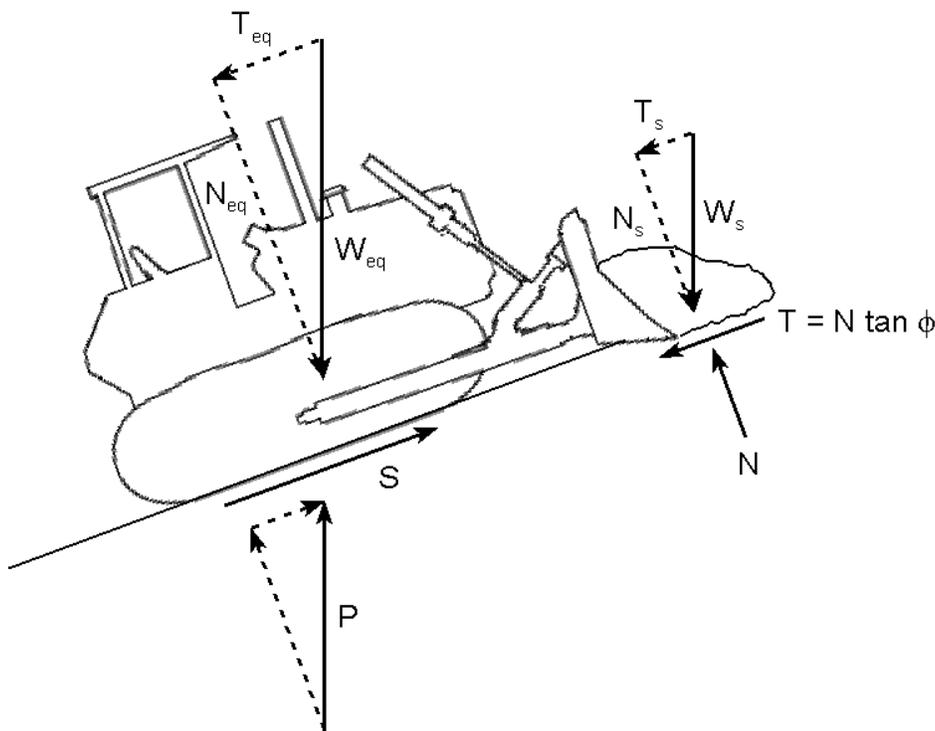


Figure 1: Forces Acting on a Bulldozer Traveling Upslope

must be considered, let us not forget why the bulldozer is on the slope. The soil being spread by the dozer imposes certain forces that should also be considered, rather than simply assumed as being negligible. As the bulldozer spreads the soil, the dozer must overcome not only the weight component of the soil acting downslope against the blade (T_s), but the shear force in the soil itself as it is spread ($N \tan N$). By examining forces parallel to the slope it can be seen that, for the system to be in equilibrium there must exist a total resisting force (in the upslope direction) acting on the bottom of the tracks equal and opposite to the forces acting downslope (i.e., the

weight components of the bulldozer (T_{eq}) and soil against the blade, and the shear force in the soil being spread). It is reasonable to assume that these forces are equally distributed to the tracks on each side of the bulldozer. It should be noted that, for convenience, the force P in Figure 1 can be taken simply as the weight of the bulldozer. In this way, the force equal and opposite to the tangential component of the bulldozer's weight is incorporated in P . The force S in Figure 1 is then equal and opposite to the sum of the weight component of the soil acting downslope against the blade plus the shear force in the soil pile

being spread. Inertia forces can also be incorporated into the S term to account for acceleration and deceleration. For example, deceleration of the dozer after backing up (or traveling in the downslope direction) would result in a force in addition to the dozer's weight component that would act downslope when the bulldozer applies the brakes. Finally, to simplify the calculations, the bulldozer blade is assumed to be straight and frictionless (i.e., the soil pile does not increase the normal force beneath the dozer tracks). This assumption essentially maximizes the shear force within the soil being spread.

Let us now focus on the system of forces acting in the geocomposite liner system itself, specifically the soil cover immediately between the bulldozer track(s) and top of the first geosynthetic component. At this point it becomes convenient to model the problem in terms of three sliding blocks, as illustrated conceptually in Figure 2, with an active block (upslope of the dozer), a passive block (downslope of the dozer), and a central block (immediately beneath the track). Since the distance between bulldozer tracks will typically be larger than the depth of the soil cover being spread over the liner system, the system of sliding blocks illustrated in Figure 2 will develop beneath each of the two bulldozer tracks. It has been noted by Druschel and Underwood (1993) that soil arching mobilizes the soil between the tracks; however, the author is not convinced of this and the method proposed here ignores this potential effect. Thus, if it is assumed that the loads imposed by the equipment are equally distributed to each track, then a solution need only be developed for one set of blocks. Alternatively, one can also consider a critical combination of loads upon one track if such a condition were of interest to the designer.

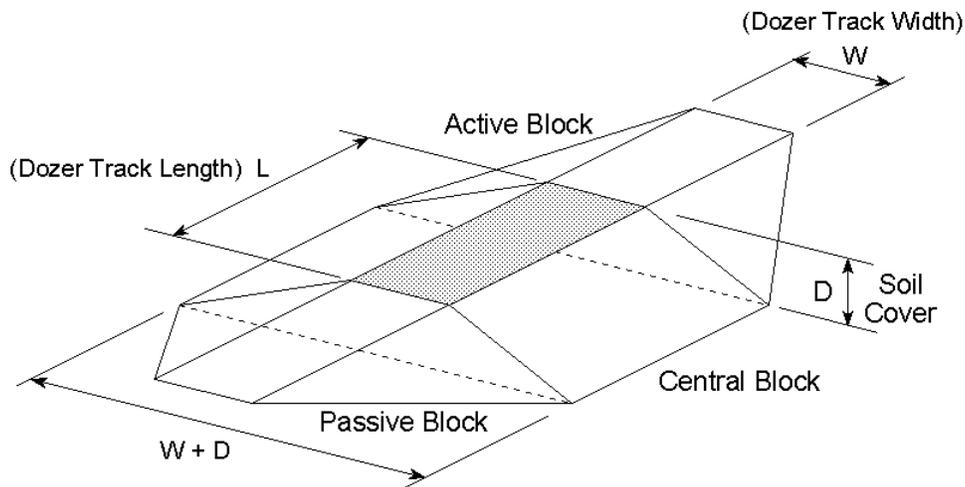


Figure 2: Sliding Blocks Beneath a Single Bulldozer Track

It is reasonable to assume that the load imposed by the bulldozer is distributed down through the soil cover to the underlying liner component, and since the depths are typically very small a simple model has been used here employing the 1(H):2(V) approximation as presented in Winterkorn and Fang (1975) and elsewhere. As shown in Figure 2, the approximation has only been applied in the cross direction, wherein the contact area at the base of the central block is equal to the product of the contact length of the dozer track (L) and the width of the track plus depth of soil cover ($W+D$). The reader can refer to Poulos and Davis (1974) for a Boussinesq model should he/she wish to do so. For such analyses to be meaningful the engineer will need to use actual weights and dimensions for specific pieces of equipment, and such information is available from the manufacturer. Once the dimensions of the base of the central block have been determined, the three dimensional blocks in Figure 2 can be replaced by three dimensional prisms as illustrated in Figure 3, which greatly simplifies the calculation of the weights of the respective blocks.

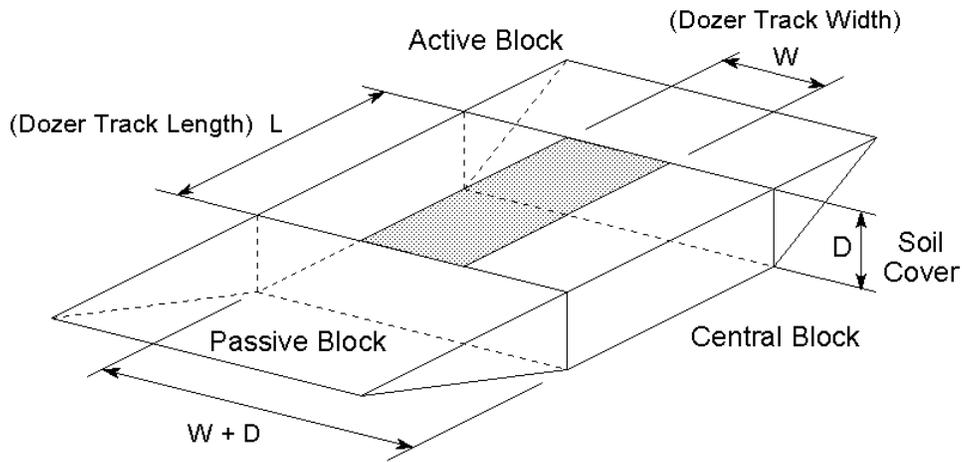


Figure 3: Prisms for Sliding Block Analysis

The independent and dependent variables in the problem will be addressed in detail in subsequent paragraphs, but before doing so let us examine some other loading conditions. Figure 4 illustrates the case of a bulldozer spreading soil while traveling downslope, and it is relevant to examine the forces that

act in this problem. From Figure 4 it can be seen that, while the same set of forces exists as in the case of travel upslope, the forces do not act in the same way. Observe that the shear force in the soil pile being spread acts in the upslope direction, while the tangential component of the weight of soil pile does not contribute to the force S beneath the dozer track, since it does not pull on the bulldozer blade. While acceleration of the bulldozer downslope is not likely to impart significant forces to the cover system, deceleration (braking) certainly does and, as previously noted, this force can be applied to the S term shown in Figure 4 as a percentage of the weight of the equipment. This system of forces would have to be compared to those that develop as the bulldozer spreads soil in the upslope direction.

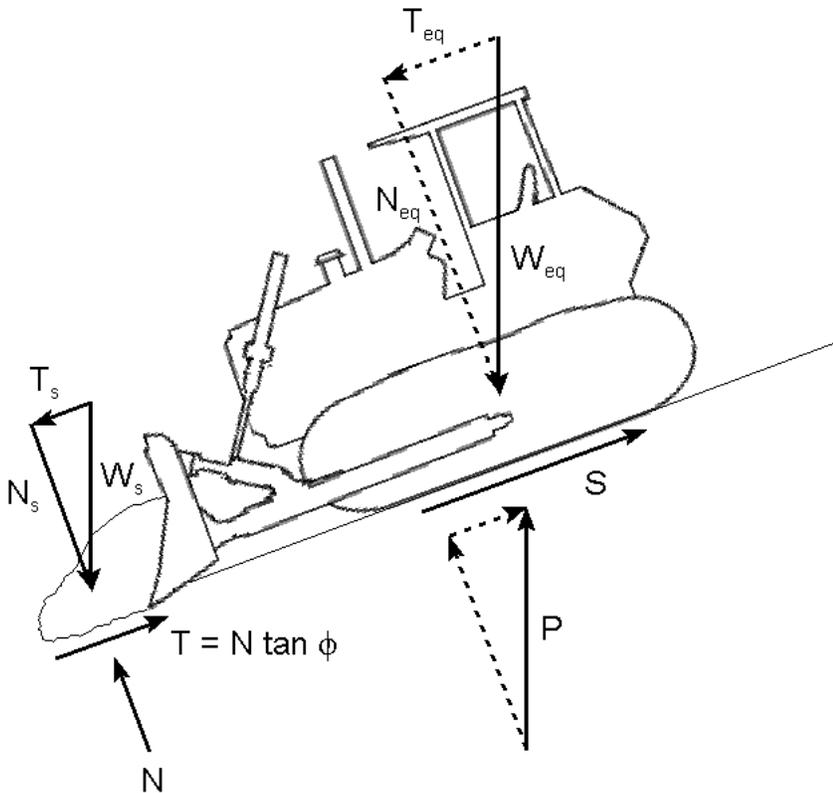


Figure 4: Forces Acting as a Bulldozer Travels Downslope

The system of forces associated with rubber tired equipment is similar, but not exactly the same as for track mounted equipment. Figure 5 illustrates the system of forces associated with a motor grader spreading soil in the upslope direction. As in the case of the bulldozer, acting downslope are the weight components of the grader and soil against the blade, and the shear force in the soil being spread. In

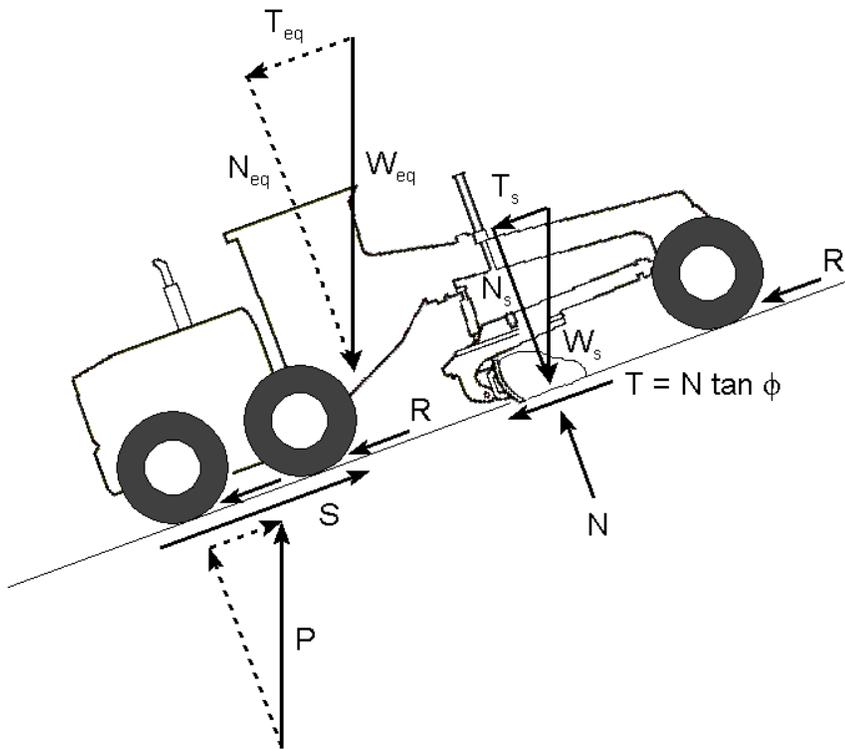


Figure 5: Forces Acting on a Motor Grader Traveling Upslope

addition, however, are the forces due to rolling resistance (R) at each of the grader's six tires. A basic discussion of rolling resistance is given by Beer and Johnston (1988) who show that the rolling resistance is a function of the radius of the tire, weight on the wheel, and coefficient of rolling resistance. Once again, by examining forces parallel to the slope it can be seen that for the grader to move upslope it must develop a total force greater than the forces acting downslope; however, this force is not distributed to each of the wheels of the grader. The force only acts against the wheels that are on actual driving axles; therefore, the number of driving axles must be established for the particular piece of equipment being analyzed. As in the case of the bulldozer, the engineer will need to obtain actual weights

and dimensions for the specific pieces of rubber tired equipment, which must now include the details of the tires on the equipment. Now the forces that the equipment imparts to the geocomposite liner system are concentrated over a smaller area than in the case of track mounted equipment, and of specific interest is the loaded section width of the tire(s) on the actual driving axles, which can frequently be obtained from the manufacturer's product literature. Figure 6 illustrates the system of sliding blocks that develops beneath a single rubber tire on a driving axle. For this case the contact area at the bottom of the tire is taken as the square of the tire's loaded section width (LSW)² and the 1(H):2(V) approximation is applied in both directions to model the distribution of the load imposed by the tire down through the soil cover to the underlying liner component, which yields the dimensions of the base of the central block. In a manner similar to that of a motor grader, the reader can develop numerical models to consider the forces imposed

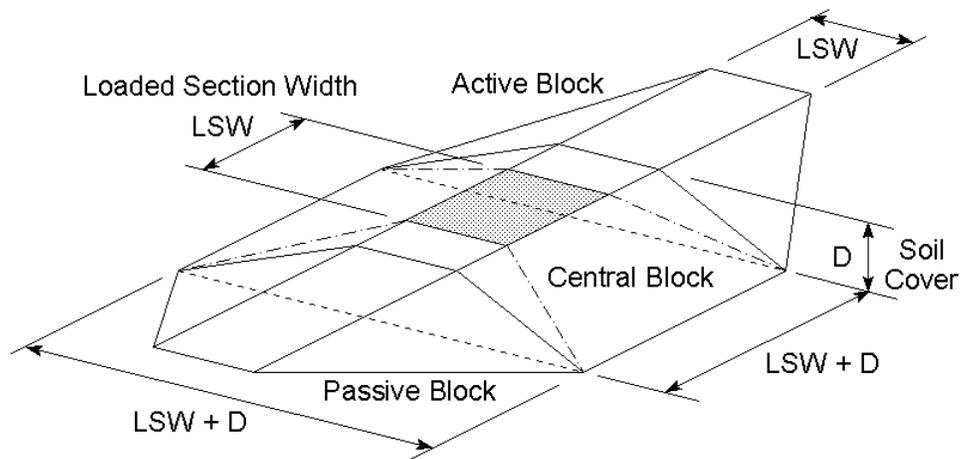


Figure 6: Sliding Blocks Beneath a Rubber Tire

by other types of rubber tired equipment, such as maintenance trucks and tractors, and the approach presented here can be extended to include compactors.

SOLUTION ALGORITHM

Having examined the various forces imposed by the equipment operating on the cover system, let us now turn our attention to the forces acting in the cover itself. Figure 7 illustrates the three sliding blocks associated with a single bulldozer track and the respective forces acting on each block. This is the same set of sliding blocks that would act beneath a single tire of a motor grader, with the only difference being in the S and P terms acting on the central block. Note that any forces acting on the sides of the blocks (parallel to the direction of travel) have been ignored. These forces will be a function of the lateral stress against the sliding blocks, the friction angle of the cover soil, and area of the sides of the respective blocks; consequently, it is anticipated that they will be relatively small considering the shallow depth of soil cover. Nevertheless, it would not be too difficult to add these forces to the system of forces acting on the blocks illustrated in Figure 7 if the reader wishes to do so. The passive, central, and active blocks are designated as blocks 1, 2, and 3 respectively, and the blocks are inclined at angles β , α (equal to the inclination angle of the slope), and θ respectively. The surface between the passive block and central block is designated as 4, and the surface between the central block and active block is designated as 5, both of which are assumed to be vertical. Note that the shear forces (T) that develop along the base of each block, as well as the interfaces between blocks, are mobilized shear strengths (denoted by the subscript m). The mobilized strength values are a function of the safety factor (F) and available strength values (soil friction angle N, interface friction angle δ , and interface adhesion a), as shown in Equations 1 and 2 of Figure 7. The side force inclination between blocks is equal to the mobilized friction angle of the cover soil, following the approach suggested by Sultan and Seed (1966). A force (T_G) is also shown

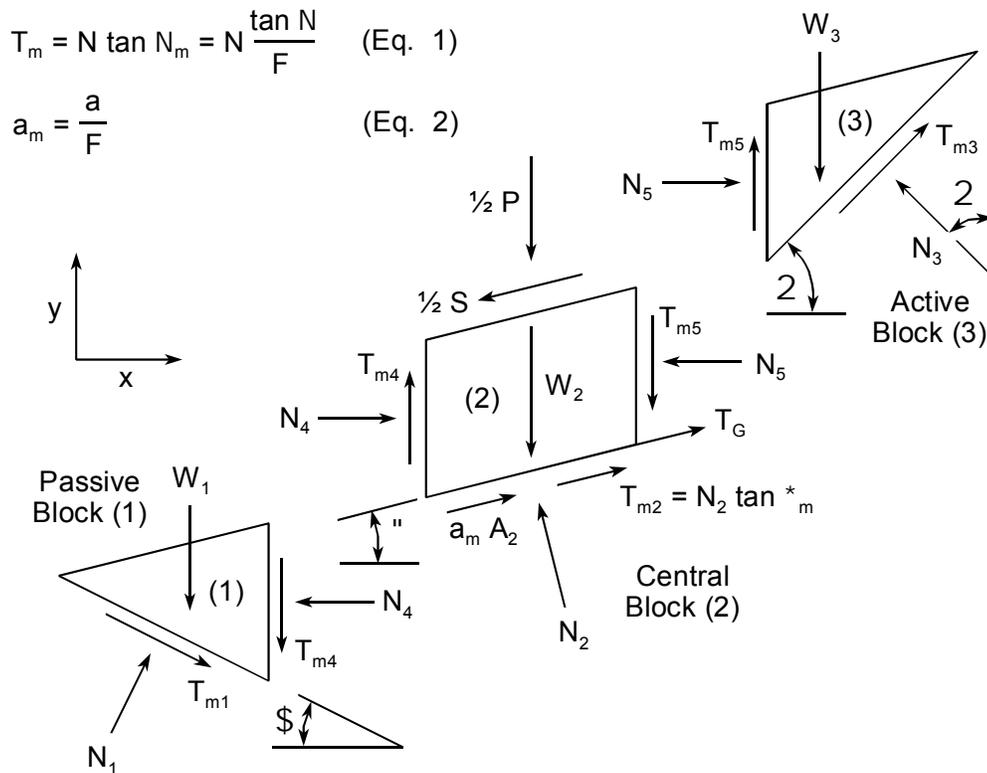


Figure 7: Sliding Blocks Beneath a Bulldozer Track

acting at the base of the central block, and this force can be set as an independent variable to model the effect of tension in geosynthetic components in the liner system beneath the soil cover. It is imperative to note, however, that the force T_G can only develop if sufficient strength exists between the soil cover and geosynthetic in immediate contact with the cover through interface friction and/or adhesion (i.e., if slippage occurs along this interface, the underlying geosynthetics can provide no contribution to the stability of the cover system). Consequently, this is a criteria that must first be checked before a value for T_G can be applied to the analysis.

There are a number of independent and dependent variables associated with the problem, as can be seen from Figure 1 through 7. The designer must first establish the type and size of the equipment that will be spreading the soil on the slope and determine (or specify) the type and amount of cover soil that will be spread at the blade. The safety factor will also depend on the unit weight and depth of the soil cover (D in Figures 2, 3, and 6), the inclination angle of the slope (α in Figure 7), and the angles that the bases of the passive and active blocks make with the horizontal (β and θ respectively in Figure 7). Regarding the depth of the cover soil, it is important to note that the depth used in the analysis should not be the final depth of the cover, but rather the depth of the first lift of soil spread by the equipment. Naturally, values for the strength parameters are also independent variables. From Figure 7 it can be seen that there are six (6) unknowns in the problem, these being the safety factor (F), and normal forces (N_{1-5}) acting on the bottom and interface surfaces of the blocks. Summing forces in the x (horizontal) and y (vertical) directions for each of the three blocks permits the development of expressions for each of the normal forces, and these equations are presented in Figure 8. Embedded in each of the equations, 3 through 8, is the unknown safety factor in the form of the mobilized strength parameters. A solution to these equations can be obtained as follows.

An initial value is assumed for the safety factor, and each of the respective normal forces are then computed using the expressions in Figure 8. It will be noted in Figure 8 that there are two expressions for the normal force (N_{5AB} and N_{5CB}) between the active and central blocks, and the initial assumption for the safety factor will generally produce two different values for that force. Obviously, the normal force can have only one value, and that value will be obtained for a unique value of safety factor. In other words, only one value for the safety factor will yield a correct solution to the equilibrium equations for any given set of independent variables, and a simple iterative process executed by means of a spreadsheet macro quickly converges on a unique value for the safety factor.

The method of halving the interval has been found to work quite effectively, and the algorithm consists of the following. Initial values are set in the spreadsheet for an upper limit and lower limit to the safety factor. For each iteration, the spreadsheet takes the safety factor as half the sum of the upper and lower limits, computes the mobilized strength values for the cover soil friction angle (N_m) and interface friction angle (δ_m) using Eq 1, and for the interface adhesion (a_m) using Eq 2, and solves for the normal forces acting on the blocks using Equations 3 through 8.² A comparison is then made of the computed values of N_{5AB} and N_{5CB} . If N_{5CB} is less than N_{5AB} , the spreadsheet copies the contents of the cell containing the safety factor (as a value) into the cell containing the upper limit (effectively halving the interval), whereupon new values are computed for the safety factor, mobilized strength values, and normal forces acting on the blocks, which constitutes the information for the next iteration. Conversely, if N_{5CB} is greater than N_{5AB} , the spreadsheet copies the contents of the cell containing the safety factor (as a value) into the cell containing the lower limit, whereupon the next iteration is performed. This procedure

² The author has made a correction to a typographical error in this sentence. In the paper as it appears in the conference proceedings, this sentence refers the reader to Fig 1 and Fig 2; however, the reference should actually be to Eq 1 and Eq 2.

converges on a safety factor quite rapidly. Figure 9 illustrates the general layout and macro commands (in Quattro-Pro) for a spreadsheet solution to the problem.³

From the equilibrium equations for the Passive Block (Block 1) :

$$N_1 = \frac{W_1}{\cos \delta - \tan N_m \sin \delta - (\sin \delta + \tan N_m \cos \delta) \tan N_m} \quad (\text{Eq 3})$$

$$N_4 = N_1 (\sin \delta + \tan N_m \cos \delta) \quad (\text{Eq 4})$$

From the equilibrium equations for the Central Block (Block 2) :

$$N_2 = \frac{W_2 + \frac{1}{2} P - (a_m A_2 + T_G - \frac{1}{2} S)(\sin \alpha - \cos \alpha \tan N_m)}{\cos \alpha + \tan^* N_m \sin \alpha + (\sin \alpha - \tan^* N_m \cos \alpha) \tan N_m} \quad (\text{Eq 5})$$

$$N_{5CB} = N_4 + N_2 (\tan^* N_m \cos \alpha - \sin \alpha) + (a_m A_2 + T_G - \frac{1}{2} S) \cos \alpha \quad (\text{Eq 6})$$

From the equilibrium equations for the Active Block (Block 3) :

$$N_3 = \frac{W_3}{\cos 2 + \tan N_m \sin 2 + (\sin 2 - \tan N_m \cos 2) \tan N_m} \quad (\text{Eq 7})$$

$$N_{5AB} = N_3 (\sin 2 - \tan N_m \cos 2) \quad (\text{Eq 8})$$

Figure 8: Equations for the Normal Forces Acting on the Blocks

The data in Figure 9 is basically self explanatory and serves as an example illustrating the results of an analysis for a Caterpillar D6D bulldozer with low ground pressure tracks pushing 1.5 cubic yards of cover soil up a 3(H):1(V) slope.

Regarding the tensile force (T_G) in Equations 5 and 6, it should be noted that the force is treated as an external force acting on the central block, not a "mobilized" strength as defined in Equations 1 or 2. The tensile force is computed as the product of the tensile strength (in dimensions of force per linear dimension, such as kN/m or lb/in) and the width of the central block ($W+D$ in Figure 2, or $LSW+D$ in Figure 6). The following approach is recommended when considering this tensile force in the analysis. Begin by computing the safety factor with the tensile strength set at zero, rather than some maximum allowable value. If an acceptable safety factor is obtained, there is no need to consider the effect of the geosynthetic. However, if the safety factor is unacceptable, input a nominal value for the tensile strength and compute the associated safety factor. Increase the tensile strength value in the analysis until the

³ A spreadsheet example illustrating the method can be downloaded at no cost from the author's web site (<http://home.earthlink.net/~dkerkes>). The spreadsheet is in Microsoft Excel format and provides a solution for the case of a bulldozer traveling in the upslope direction while spreading cover soil over the liner system.

safety factor equals or slightly exceeds the minimum allowable safety factor, and then compare this "required tensile strength" to the material's allowable strength value. It is the author's opinion that this provides a more reasonable way of evaluating the interaction of the components in the cover system, enabling the designer to consider the respective levels of strain in each of the contributing components.

LIMITATIONS

The basic method proposed herein, though commonly used, has some inherent limitations that the designer should be aware of. The approach employs the limit equilibrium method; however, as is typically the case in sliding block analyses of this type, the solution considers only the conditions for force equilibrium. That is to say, the conditions of moment equilibrium are never examined.

A problem sometimes (though not frequently) encountered with the approach is the inability of the algorithm to converge on a solution. Modifying the macro to allow the engineer to examine the results one iteration at a time will often assist in identifying the trouble, and a slight modification to one of the variables will sometimes lead to an acceptable solution. The macro commands shown in Figure 9 contain a check that alerts the user when the spreadsheet encounters a problem with convergence.

Occasionally a negative value is computed for one of the normal forces acting on the blocks, most commonly the active block. While it is appropriate to check the equations in the spreadsheet for errors, the result is not necessarily due to an error in the algorithm. A negative value indicates that a tensile force is required to satisfy the conditions of equilibrium; consequently, such a result must be interpreted as unacceptable since no such tensile forces can in fact develop. For a solution to be acceptable, the normal forces cannot be negative. The macro commands shown in Figure 9 contain a check that alerts the user when a solution is obtained that produces a negative value for any one of the five normal forces.

It is relevant to note that the issue of strain compatibility is not addressed in any limit equilibrium method. Consequently, the results of the analysis proposed herein should be evaluated in concert with a review of the stress-strain curves for each component interface to determine if the mobilized strength values computed in the analysis are reasonable. In lieu of this approach, many members of the profession simply recommend the use of residual strength values.

The method does not directly yield a "minimum safety factor," since the safety factor is a function of the angles α , β , and θ , as well as the other variables previously noted. While the angle α is generally fixed in the analysis as the inclination of the slope, β and θ are independent variables that will affect the safety factor; thus, the designer will need to try several combinations for these angles.

SUMMARY

It is generally accepted by most designers that strength data used for final design should be obtained from laboratory tests on the materials actually intended for use in construction and should not simply be taken from the literature. Similarly, it would be inappropriate to ignore the details associated with the actual equipment planned for use in construction of the liner system, since the loads produced by such equipment frequently constitute the most severe conditions that the materials will experience during their service life. The method proposed here, which can be implemented using ordinary spreadsheet software, is a practical approach that considers many of the variables associated with the problem and allows the effect of these variables to be examined by extending a solution technique that has been used by the profession for many years in slope stability analyses.

Sliding Block Analysis with Surface Loads (P & S) and Geotextile Tensile Force (Tg)
 Bulldozer Spreading Soil Upslope

Safety Factor F = 1.309

Unit Weight of Cover Soil 15.71 kN/cu m
 Depth of Soil Cover (D) 0.3 m

Errors ? None

CAT D6D LGP Dozer 170 kN (total weight)
 Track Length (L) 2.90 m
 Track Width (W) 0.91 m
 Width at Interface (W+D) 1.21 m
 Contact Area at Interface 3.51 sq. m

$\beta = 15.00 \text{ deg} = 0.262 \text{ rad}$
 $\alpha = 18.43 \text{ deg} = 0.322 \text{ rad}$
 $\theta = 60.00 \text{ deg} = 1.047 \text{ rad}$

P = 85.0 kN (per track)
 S = 7.8 kN (per track)
 Unit Tension 7.0 kN/m (geosynthetic)
 Tg = 8.5 kN (Width at Interface * Unit Tension)

	Available (specified)	Mobilized (computed)
Soil Cover Friction Angle	30.0 deg = 0.524 rad	23.8 deg = 0.416 rad
Interface Friction Angle	22.0 deg = 0.384 rad	17.2 deg = 0.299 rad
Interface Adhesion	0.0 kN/sq m	0.0 kN/sq m

Block	Weight	Bottom Surface Area
Passive (1)	1.58 kN	
Central (2)	16.54 kN	3.51 sq m
Active (3)	0.68 kN	

$N_{(1)} = 2.88 \text{ kN}$ Figure 8: Eq. 3
 $N_{(2)} = 96.17 \text{ kN}$ Figure 8: Eq. 5
 $N_{(3)} = 0.58 \text{ kN}$ Figure 8: Eq. 7
 $N_{(4)} = 1.97 \text{ kN}$ Figure 8: Eq. 4
 $N_{(5)CB} = 0.37 \text{ kN}$ Figure 8: Eq. 6
 $N_{(5)AB} = 0.38 \text{ kN}$ Figure 8: Eq. 8

Method of Halving the Interval (Macro commands for Quattro-Pro)

```
MACRO \C {LET Counter,0}{LET Upper Limit,10}{LET Lower Limit,0}{CALC}{BRANCH \X}
MACRO \X {IF @ABS(N(5)CB - N(5)AB)<0.1}{BRANCH \N}
{LET Counter,Counter+1}{IF Counter>35}{LET Message,"Convergence Error"}{QUIT}
{IF N(5)CB < N(5)AB}{BLOCKVALUES F,Upper Limit}{BRANCH \X}
{BLOCKVALUES F,Lower Limit}{BRANCH \X}
MACRO \N {IF N(1)>0}{IF N(2)>0}{IF N(3)>0}{IF N(4)>0}{IF N(5)CB>0}{LET Message,"None"}{QUIT}
{LET Message,"Side Force Error"}{QUIT}
```

Figure 9: Spreadsheet Example

REFERENCES

- Beer, F. P., & Johnston, E. R. (1988) Vector Mechanics for Engineers - Statics, McGraw-Hill Book Co., New York
- Corcoran, G. T. & McKelvey, J. A. (1995) "Stability of Soil Layers on Compound Geosynthetic Slopes," Proceedings of Waste Tech '95, New Orleans, LA, pp 301-314
- Druschel, S. J. & Underwood, E. R. (1993), "Design of Lining and Cover System Sideslopes," Proceedings of Geosynthetics '93, Vancouver, Canada, pp 1341-1355
- Koerner, R. M., & Soong, T-Y, (1998) "Analysis and Design of Veneer Cover Soils," Proceedings of the Sixth International Conference on Geosynthetics, Atlanta, GA, pp 1-23
- McKelvey, J. A. (1991), "Consideration of Equipment Loading in Geosynthetic Lined Slope Design," Proceedings of the Eighth International Conference on Computer Methods and Advances in Geomechanics, Morgantown, WV, pp 1371-1377
- McKelvey, J. A. & Deutsch, W. L. (1991), "The Effect of Equipment Loading and Tapered Cover Soil Layers on Geosynthetic Lined Landfill Slopes," Proceedings of the Fourteenth Madison Waste Conference, Madison, WI, pp 395-411
- Poulos, H. G., & Davis, E. H., (1974) Elastic Solutions for Soil and Rock Mechanics, John Wiley & Sons, New York
- Sultan, H. A., & Seed, H. B., (1966) "Stability of Sloping Core Earth Dams," Proceedings of the ASCE Conference on Stability and Performance of Slopes and Embankments, Berkeley, CA, pp 51-73
- Winterkorn, H. F., & Fang, H-Y, (1975) Foundation Engineering Handbook, Van Nostrand Reinhold Co., New York